

Self consistent propagation of hyperons and antikaons in nuclear matter based on relativistic chiral SU(3) dynamics

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We evaluate the antikaon spectral density in isospin symmetric nuclear matter. The in-medium antikaon-nucleon scattering process and the antikaon propagation is treated in a self consistent and relativistic manner where a maximally scheme-independent formulation is derived by performing a partial density resummation in terms of the free-space antikaon-nucleon scattering amplitudes. The latter amplitudes are taken from a relativistic and chiral coupled-channel SU(3) approach which includes s-, p- and d-waves systematically. Particular care is taken on the proper evaluation of the in-medium mixing of the partial waves. Our analysis establishes a rich structure of the antikaon spectral function with considerable strength at small energies. At nuclear saturation density we predict attractive mass shifts for the $\Lambda(1405)$, $\Sigma(1385)$ and $\Lambda(1520)$ of about 130 MeV, 60 MeV and 100 MeV respectively. The hyperon states are found to exhibit at the same time an increased decay width of about 150 MeV for the s-wave $\Lambda(1405)$, 70 MeV for the p-wave $\Sigma(1385)$ and 100 MeV for the d-wave $\Lambda(1520)$ resonance.

1 Introduction

A good understanding of the antikaon spectral function in nuclear matter is required for the description of K^- -atoms [1,2] and the subthreshold production of kaons in heavy ion reactions [3]. An exciting consequence of a significantly reduced effective K^- mass could be that kaons condense in the interior of neutron stars [4,5]. The ultimate goal is to relate the in-medium spectral function of kaons with the anticipated chiral symmetry restoration at high baryon density. To unravel quantitative constraints on the kaon spectral functions from

subthreshold kaon production data of heavy-ion reactions requires transport model calculations which are performed extensively by various groups [6–9]. The next generation of transport codes which are able to incorporate more consistently particles with finite width are being developed [10–13]. This is of considerable importance when dealing with antikaons which are poorly described by a quasi-particle ansatz [14,15].

There has been much theoretical effort to access the properties of kaons in nuclear matter [16–20,14,15,21]. An antikaon spectral function with support at energies smaller than the free-space kaon mass was already anticipated in the 70’s by the many K-matrix analyses of the antikaon-nucleon scattering process (see e.g. [22]) which predicted considerable attraction in the subthreshold scattering amplitudes. This leads in conjunction with the low-density theorem [23,16] to an attractive antikaon spectral function in nuclear matter. Nevertheless, the quantitative evaluation of the antikaon spectral function is still an outstanding problem. The challenge is first to develop an improved understanding of the vacuum antikaon-nucleon scattering process, in particular reliable subthreshold antikaon-nucleon scattering amplitudes are required, and secondly, to evaluate the antikaon spectral function in an improved many-body treatment.

The antikaon-nucleon scattering is complicated due to the open inelastic $\pi\Sigma$ and $\pi\Lambda$ channels and the presence of the s-wave $\Lambda(1405)$ and p-wave $\Sigma(1385)$ resonances just below and the d-wave $\Lambda(1520)$ resonance not too far above the antikaon-nucleon threshold. Since the low-energy data on antikaon-nucleon scattering are not too precise the required subthreshold extrapolation is favorably performed imposing constraints from causality, covariance and chiral symmetry. First intriguing works in this direction can be found in [24,25] where the low-energy antikaon-proton scattering data were successfully described in terms of s-wave coupled SU(3) channels. These works are based on a cut-off regularized Lippmann-Schwinger equation where the potential is matched phenomenologically with the leading tree-level terms of the chiral Lagrangian. The fact that the subthreshold amplitudes of [24] and [25] differ significantly triggered the recent work [26] which systematically considers s-, p-, and d-waves. The χ -BS(3) approach of [26] constitutes a considerably improved chiral scheme more consistent with covariance, crossing symmetry and the chiral counting concept. The amplitudes obtained in that scheme are particularly well suited for an application to the nuclear kaon dynamics, because it was demonstrated that they are approximately crossing symmetric in the sense that the KN and $\bar{K}N$ amplitudes smoothly match at subthreshold energies. Moreover those amplitudes describe the kaon and antikaon-nucleon scattering processes up to much higher energies, quantitatively up to $p_{\text{lab.}} \simeq 500$ MeV and qualitatively to about $p_{\text{lab.}} \simeq 1000$ MeV. Therefore we believe that those amplitudes, which are of central importance for the nuclear kaon dynamics, will lead to rather reliable results for the propagation properties of kaons in

dense nuclear matter.

As was pointed out in [14] the realistic evaluation of the antikaon self energy in nuclear matter requires a self consistent scheme. In particular the feedback effect of an in-medium modified antikaon spectral function on the antikaon-nucleon scattering process was found to be important for the $\Lambda(1405)$ resonance structure in nuclear matter. In this work we apply the newly developed χ -BS(3) approach of [26] to antikaon propagation in nuclear matter in a self consistent manner. We develop a novel, maximally scheme-independent and covariant framework in which self consistency is implemented in terms of the vacuum meson-nucleon scattering amplitudes. Our scheme considers for the first time the in-medium mixing of s-, p- and d-waves. Besides deriving a realistic antikaon spectral function we obtain the first quantitative results on the in-medium structure of the p-wave $\Sigma(1385)$ and the d-wave $\Lambda(1520)$ resonances.

2 Self consistent nuclear antikaon dynamics

In this section we introduce the self consistent and relativistic many-body framework required for the evaluation of the antikaon propagation in nuclear matter. First we recall the vacuum on-shell antikaon-nucleon scattering amplitude

$$\langle \bar{K}^j(\bar{q}) N(\bar{p}) | T | \bar{K}^i(q) N(p) \rangle = (2\pi)^4 \delta^4(q + p - \bar{q} - \bar{p}) \times \bar{u}(\bar{p}) T_{\bar{K}N \rightarrow \bar{K}N}^{ij}(\bar{q}, \bar{p}; q, p) u(p), \quad (1)$$

where $\delta^4(\dots)$ guarantees energy-momentum conservation and $u(p)$ is the nucleon isospin-doublet spinor. Note also $\bar{K} = (K^-, \bar{K}^0)$. The vacuum scattering amplitude is decomposed into its isospin channels

$$T_{\bar{K}N \rightarrow \bar{K}N}^{ij}(\bar{q}, \bar{p}; q, p) = T_{\bar{K}N}^{(0)}(\bar{k}, k; w) P_{(I=0)}^{ij} + T_{\bar{K}N}^{(1)}(\bar{k}, k; w) P_{(I=1)}^{ij}, \\ P_{(I=0)}^{ij} = \frac{1}{4} \left(\delta^{ij} 1 + (\vec{\tau})^{ij} \vec{\tau} \right), \quad P_{(I=1)}^{ij} = \frac{1}{4} \left(3 \delta^{ij} 1 - (\vec{\tau})^{ij} \vec{\tau} \right), \quad (2)$$

where q, p, \bar{q}, \bar{p} are the initial and final kaon and nucleon 4-momenta and

$$w = p + q = \bar{p} + \bar{q}, \quad k = \frac{1}{2}(p - q), \quad \bar{k} = \frac{1}{2}(\bar{p} - \bar{q}). \quad (3)$$

In quantum field theory the scattering amplitudes $T_{\bar{K}N}^{(I)}$ follow as the solution of the Bethe-Salpeter matrix equation

$$T(\bar{k}, k; w) = K(\bar{k}, k; w) + \int \frac{d^4 l}{(2\pi)^4} K(\bar{k}, l; w) G(l; w) T(l, k; w) ,$$

$$G(l; w) = -i S_N(\tfrac{1}{2} w + l) D_{\bar{K}}(\tfrac{1}{2} w - l) , \quad (4)$$

in terms of the Bethe-Salpeter kernel $K(\bar{k}, k; w)$, the free space nucleon propagator $S_N(p) = 1/(\not{p} - m_N + i\epsilon)$ and kaon propagator $D_K(q) = 1/(q^2 - m_K^2 + i\epsilon)$. Following [26] we neglect here self energy corrections in the nucleon and kaon propagators. In a chiral scheme such effects are of subleading order. The Bethe-Salpeter equation (4) implements properly Lorentz invariance and unitarity for the two-body scattering process. The generalization of (4) to a coupled-channel system is straightforward.

The antikaon-nucleon scattering process is readily generalized from the vacuum to the nuclear matter case. In compact notation we write

$$\mathcal{T} = \mathcal{K} + \mathcal{K} \cdot \mathcal{G} \cdot \mathcal{T} , \quad \mathcal{T} = \mathcal{T}(\bar{k}, k; w, u) , \quad \mathcal{G} = \mathcal{G}(l; w, u) , \quad (5)$$

where the in-medium scattering amplitude $\mathcal{T}(\bar{k}, k; w, u)$ and the two-particle propagator $\mathcal{G}(l; w, u)$ depend now on the 4-velocity u_μ characterizing the nuclear matter frame. For nuclear matter moving with a velocity \vec{v} one has

$$u_\mu = \left(\frac{1}{\sqrt{1 - \vec{v}^2/c^2}}, \frac{\vec{v}/c}{\sqrt{1 - \vec{v}^2/c^2}} \right) , \quad u^2 = 1 . \quad (6)$$

We emphasize that (5) is properly defined from a Feynman diagrammatic point of view even in the case where the in-medium scattering process is no longer well defined due to a broad antikaon spectral function. In this work we do not consider medium modifications of the interaction kernel, i.e. we approximate $\mathcal{K} = K$. We exclusively study the effect of a in-medium modified two-particle propagator \mathcal{G}

$$\Delta S_N(p, u) = -2\pi i \Theta(p \cdot u) \delta(p^2 - m_N^2) (\not{p} + m_N) \Theta(k_F^2 + m_N^2 - (u \cdot p)^2) ,$$

$$\mathcal{S}_N(p, u) = S_N(p) + \Delta S_N(p, u) , \quad \mathcal{D}_{\bar{K}}(q, u) = \frac{1}{q^2 - m_K^2 - \Pi_{\bar{K}}(q, u)} ,$$

$$\mathcal{G}(l; w, u) = -i \mathcal{S}_N(\tfrac{1}{2} w + l, u) \mathcal{D}_{\bar{K}}(\tfrac{1}{2} w - l, u) , \quad (7)$$

where the Fermi momentum k_F parameterizes the nucleon density ρ with

$$\rho = 2 \text{tr} \gamma_0 \int \frac{d^4 p}{(2\pi)^4} i \Delta S_N(p, u) = \frac{2 k_F^3}{3 \pi^2 \sqrt{1 - \vec{v}^2/c^2}} . \quad (8)$$

In the rest frame of the bulk with $u_\mu = (1, \vec{0})$ one recovers with (8) the standard result $\rho = 2k_F^3/(3\pi^2)$. In this work we also refrain from including nucleonic correlation effects. Such effects have been estimated to be small at moderate densities [20] but should nevertheless be subject of a more complete consideration. The antikaon self energy $\Pi_{\bar{K}}(q, u)$ is evaluated self consistently in terms of the in-medium scattering amplitudes $\mathcal{T}_{\bar{K}N}^{(I)}(\vec{k}, k; w, u)$

$$\begin{aligned}\Pi_{\bar{K}}(q, u) &= -2 \text{tr} \int \frac{d^4 p}{(2\pi)^4} i \Delta S_N(p, u) \bar{\mathcal{T}}_{\bar{K}N}(\tfrac{1}{2}(p-q), \tfrac{1}{2}(p-q); p+q, u), \\ \bar{\mathcal{T}}_{\bar{K}N} &= \frac{1}{4} \mathcal{T}_{\bar{K}N}^{(I=0)} + \frac{3}{4} \mathcal{T}_{\bar{K}N}^{(I=1)}.\end{aligned}\quad (9)$$

In order to solve the self consistent set of equations (5,7,9) it is convenient to rewrite the scattering amplitude as follows

$$\mathcal{T} = K + K \cdot \mathcal{G} \cdot \mathcal{T} = T + T \cdot \Delta \mathcal{G} \cdot \mathcal{T}, \quad \Delta \mathcal{G} = \mathcal{G} - G, \quad (10)$$

where $T = K + K \cdot G \cdot T$ is the vacuum scattering amplitude. The idea is to start with a set of tabulated coupled-channel scattering amplitudes T in terms of which self consistency is implemented. Thus the vacuum interaction kernel K need not to be specified.

Coupled channel effects, which are known to be important for $\bar{K}N$ -scattering, are included by assigning \mathcal{T} , \mathcal{G} and K the appropriate matrix structures. Here we use the convention of [26]. For example the isospin zero loop matrix $\Delta \mathcal{G}^{(I=0)}$ reads

$$\Delta \mathcal{G}^{(I=0)} = \begin{pmatrix} \Delta \mathcal{G}_{\bar{K}N} & 0 & 0 & 0 \\ 0 & \Delta \mathcal{G}_{\pi\Sigma} & 0 & 0 \\ 0 & 0 & \Delta \mathcal{G}_{\pi\Lambda} & 0 \\ 0 & 0 & 0 & \Delta \mathcal{G}_{K\Xi} \end{pmatrix}. \quad (11)$$

In this work we consider exclusively the effect of the in-medium modified $\bar{K}N$ channel. Ultimately it would be desirable to include also the effects of the in-medium modified $\pi\Sigma$, $\pi\Lambda$ and $K\Xi$ channels. Since this requires a realistic in-medium pion propagator to be determined also by a self consistent scheme [27], we will consider such effects in a separate work. Here we approximate $\Delta \mathcal{G}_{\pi\Sigma} = 0$ and $\Delta \mathcal{G}_{\pi\Lambda} = 0$. Since the K^+ spectral density is affected only moderately by a nuclear environment we assume $\Delta \mathcal{G}_{K\Xi} = 0$ also. With these assumptions the coupled-channel problem reduces to a single channel problem if rewritten in terms of the vacuum $\bar{K}N$ amplitude $T_{\bar{K}N \rightarrow \bar{K}N}$:

$$\begin{aligned} \mathcal{T}_{\bar{K}N \rightarrow \bar{K}N}^{(I)} &= T_{\bar{K}N \rightarrow \bar{K}N}^{(I)} + T_{\bar{K}N \rightarrow \bar{K}N}^{(I)} \cdot \Delta \mathcal{G}_{\bar{K}N} \cdot \mathcal{T}_{\bar{K}N \rightarrow \bar{K}N}^{(I)} , \\ \Delta \mathcal{G}_{\bar{K}N} &= \mathcal{G}_{\bar{K}N} - G_{\bar{K}N} . \end{aligned} \quad (12)$$

We point out that the self consistent set of equations (7,9,12) is now completely determined by the vacuum amplitudes $T_{\bar{K}N \rightarrow \bar{K}N}^{(I)}$. Note that even though our antikaon spectral function is a functional of the elastic $\bar{K}N \rightarrow \bar{K}N$ scattering process only, the inelastic channels $\bar{K}N \rightarrow X$ are nevertheless affected by $\Delta \mathcal{G}_{\bar{K}N}$. We find:

$$\begin{aligned} \mathcal{T}_{\bar{K}N \rightarrow X}^{(I)} &= T_{\bar{K}N \rightarrow X}^{(I)} + T_{\bar{K}N \rightarrow \bar{K}N}^{(I)} \cdot \Delta \mathcal{G}_{\bar{K}N} \cdot \mathcal{T}_{\bar{K}N \rightarrow X}^{(I)} , \\ \mathcal{T}_{Y \rightarrow X}^{(I)} &= T_{Y \rightarrow X}^{(I)} + T_{Y \rightarrow \bar{K}N}^{(I)} \cdot \Delta \mathcal{G}_{\bar{K}N} \cdot \mathcal{T}_{\bar{K}N \rightarrow X}^{(I)} , \end{aligned} \quad (13)$$

where $X, Y \neq \bar{K}N$. In (13) we derived the effect of $\Delta \mathcal{G}_{\bar{K}N}$ on the reaction $X \rightarrow Y$ with $X, Y \neq \bar{K}N$.

In the following section we will outline how to solve the self consistent set of equations (7,9,12) by introducing an appropriate projector algebra. Our scheme takes into account the medium induced mixing of partial waves usually neglected in conventional G-matrix calculations (see e.g. [21]).

3 Projector algebra method in nuclear matter

We proceed by briefly recalling the coupled-channel theory of [26]. The scattering amplitudes were obtained in a relativistic chiral $SU(3)$ approach which includes s-, p- and d-waves systematically. The parameters were successfully adjusted to reproduce the low-energy pion-nucleon and K^\pm -proton scattering data. The scattering amplitudes were decomposed systematically into covariant projectors $Y_n^{(\pm)}$ with good angular momentum $J = n + \frac{1}{2}$:

$$\begin{aligned} T^{(I)}(\bar{k}, k; w) &= \sum_{n=0}^{\infty} Y_n^{(+)}(\bar{q}, q; w) M^{(I,+)}(\sqrt{s}, n) \\ &+ \sum_{n=0}^{\infty} Y_n^{(-)}(\bar{q}, q; w) M^{(I,-)}(\sqrt{s}, n) \end{aligned} \quad (14)$$

where $w^2 = s$ and $k = \frac{1}{2}(p - q)$ and $\bar{k} = \frac{1}{2}(\bar{p} - \bar{q})$. The representation (14) implies a particular off-shell behavior of the scattering amplitude, which was optimized as to simplify the task of solving the covariant Bethe-Salpeter equation. This is legitimate because the off-shell structure of the scattering amplitude is not determined by the two-body scattering processes in any case and moreover reflects the choice of meson and baryon interpolation fields only

[28,29]. The leading projectors relevant for the $J = \frac{1}{2}$ and $J = \frac{3}{2}$ channels are $Y_0^{(+)}$ (s-wave), $Y_0^{(-)}$ (p-wave) and $Y_1^{(+)}$ (p-wave), $Y_1^{(-)}$ (d-wave) where:

$$\begin{aligned} Y_0^{(\pm)}(\bar{q}, q; w) &= \frac{1}{2} \left(\frac{\psi}{\sqrt{w^2}} \pm 1 \right), \\ Y_1^{(\pm)}(\bar{q}, q; w) &= \frac{3}{2} \left(\frac{\psi}{\sqrt{w^2}} \pm 1 \right) \left(\frac{(\bar{q} \cdot w)(w \cdot q)}{w^2} - (\bar{q} \cdot q) \right) \\ &\quad - \frac{1}{2} \left(\not{q} - \frac{w \cdot \bar{q}}{w^2} \psi \right) \left(\frac{\psi}{\sqrt{w^2}} \mp 1 \right) \left(\not{q} - \frac{w \cdot q}{w^2} \psi \right). \end{aligned} \quad (15)$$

For instance the projector $Y_1^{(-)}$ probes the d-wave $\Lambda(1520)$ resonance and $Y_1^{(+)}$ the p-wave $\Sigma(1385)$ resonance. For more details on the construction and the properties of these projectors we refer to [26].

In the following we will solve the self consistent set of equations (12,7,9) with $n = 0, 1$ in (14). The solution of the Bethe-Salpeter equation in nuclear matter is considerably complicated by the fact that the partial waves, which decouple in the vacuum, start to mix in the medium. This manifests itself in the presence of further tensor structures in the scattering amplitude not required in the vacuum. For example, if one starts to iterate the Bethe-Salpeter equation (12) in terms of the vacuum amplitudes $M^{(I,\pm)}(\sqrt{s}, n)$ incorporating $\Delta\mathcal{G}_{\bar{K}N}$ as determined by (9) with $\mathcal{T}_{\bar{K}N} = T_{\bar{K}N}$ one finds that $\mathcal{T}_{\bar{K}N}$ can no longer be decomposed into the projectors $Y_n^{(\pm)}(\bar{q}, q; w)$. The in-medium solution of the Bethe-Salpeter equation requires a more general ansatz for the scattering amplitude.

We found that the following ansatz solves the self consistent set of equations:

$$\begin{aligned} \mathcal{T}(\bar{k}, k; w, u) &= \mathcal{T}(w, u) + \mathcal{T}^\nu(w, u) q_\nu + \bar{q}_\mu \bar{\mathcal{T}}^\mu(w, u) + \bar{q}_\mu \mathcal{T}^{\mu\nu}(w, u) q_\nu, \\ \bar{\mathcal{T}}^\mu &= \sum_{j=3}^8 \left(\mathcal{M}_{[1j]}^{(p)} \bar{P}_{[1j]}^\mu + \mathcal{M}_{[2j]}^{(p)} \bar{P}_{[2j]}^\mu \right), \quad \mathcal{T}^\mu = \sum_{j=3}^8 \left(\mathcal{M}_{[j1]}^{(p)} P_{[j1]}^\mu + \mathcal{M}_{[j2]}^{(p)} P_{[j2]}^\mu \right), \\ \mathcal{T} &= \sum_{i,j=1}^2 \mathcal{M}_{[ij]}^{(p)} P_{[ij]}, \quad \mathcal{T}^{\mu\nu} = \sum_{i,j=3}^8 \mathcal{M}_{[ij]}^{(p)} P_{[ij]}^{\mu\nu} + \sum_{i,j=1}^2 \mathcal{M}_{[ij]}^{(q)} Q_{[ij]}^{\mu\nu}, \end{aligned} \quad (16)$$

where we introduced appropriate projectors $P_{[ij]}(w, u)$ which generalize the vacuum projectors $Y_n^{(\pm)}(\bar{q}, q; w)$. The matrix valued functions $\mathcal{M}_{[ij]}^{(p,q)}(w, u)$ are scalar and therefore depend only on $s = w^2$ and $w \cdot u$. The set of projectors is constructed to satisfy the convenient algebra:

$$P_{[ik]} \cdot P_{[lj]} = \delta_{kl} P_{[ij]}, \quad P_{[ik]}^\mu \bar{P}_{[lj]}^\nu = \delta_{kl} P_{[ij]}^{\mu\nu}, \quad \bar{P}_{[ik]}^\mu g_{\mu\nu} P_{[lj]}^\nu = \delta_{kl} P_{[ij]},$$

$$\begin{aligned}
Q_{[ik]}^{\mu\alpha} g_{\alpha\beta} P_{[lj]}^{\beta\nu} &= 0 = P_{[ik]}^{\mu\alpha} g_{\alpha\beta} Q_{[lj]}^{\beta\nu}, & Q_{[ik]}^{\mu\alpha} g_{\alpha\beta} P_{[lj]}^{\beta} &= 0 = \bar{P}_{[ik]}^{\alpha} g_{\alpha\beta} Q_{[lj]}^{\beta\nu}, \\
Q_{[ik]}^{\mu\alpha} g_{\alpha\beta} Q_{[lj]}^{\beta\nu} &= \delta_{kl} Q_{[ij]}^{\mu\nu}, & P_{[ik]}^{\mu\alpha} g_{\alpha\beta} P_{[lj]}^{\beta\nu} &= \delta_{kl} P_{[ij]}^{\mu\nu}.
\end{aligned} \tag{17}$$

The projector algebra (17) properly implements the coupling of the partial waves in the medium. Consider for example the projectors $P_{[ij]}(w, u)$ for $i, j = 1, 2$,

$$\begin{aligned}
P_{[11]} &= +Y_0^{(+)}, & P_{[12]} &= -Y_0^{(+)} \frac{\gamma \cdot u}{\sqrt{1 - (w \cdot u)^2/w^2}} Y_0^{(-)}, \\
P_{[22]} &= -Y_0^{(-)}, & P_{[21]} &= -Y_0^{(-)} \frac{\gamma \cdot u}{\sqrt{1 - (w \cdot u)^2/w^2}} Y_0^{(+)},
\end{aligned} \tag{18}$$

which demonstrate that the vacuum projectors start to mix in the medium due to the presence of the matter 4-velocity u_μ . If we neglected the $J = \frac{3}{2}$ amplitudes $M_1^{(\pm)}$ the Bethe-Salpeter equation is solved with the restricted set of projectors $P_{[ij]}$ given in (18). Explicit results for the remaining projectors are presented in Appendix A. In particular we find

$$Y_1^{(+)} = -3 \bar{q}_\mu \left(Q_{[11]}^{\mu\nu} + P_{[77]}^{\mu\nu} \right) q_\nu, \quad Y_1^{(-)} = +3 \bar{q}_\mu \left(Q_{[22]}^{\mu\nu} + P_{[88]}^{\mu\nu} \right) q_\nu. \tag{19}$$

We conclude that in the space of $P_{[ij]}$ -projectors all considered partial waves couple whereas in the $Q_{[ij]}$ -projector space only the two $J = \frac{3}{2}$ waves couple. This reflects the fact that the four polarizations of a spin $\frac{3}{2}$ fermion are no longer degenerate in a nuclear environment. In analogy to a spin 1 boson with one longitudinal and two degenerate transverse modes one expects two independent modes for a spin $\frac{3}{2}$ fermion in nuclear matter also. We refer to these modes as P -space and Q -space states.

With the ansatz (16) the antikaon-nucleon propagator $\Delta\mathcal{G}_{\bar{K}N}$ translates into the set of loop functions

$$\begin{aligned}
\int \frac{d^4 l}{(2\pi)^4} \Delta\mathcal{G}(l - \tfrac{1}{2}w; w, u) &= \sum_{i,j=1}^2 \Delta J_{[ij]}^{(p)}(w, u) P_{[ij]}(w, u), \\
\int \frac{d^4 l}{(2\pi)^4} l^\mu \Delta\mathcal{G}(l - \tfrac{1}{2}w; w, u) &= \sum_{j=3}^8 \left(\Delta J_{[j1]}^{(p)}(w, u) P_{[j1]}^\mu(w, u) + \Delta J_{[j2]}^{(p)}(w, u) P_{[j2]}^\mu(w, u) \right) \\
&= \sum_{j=3}^8 \left(\Delta J_{[1j]}^{(p)}(w, u) \bar{P}_{[1j]}^\mu(w, u) + \Delta J_{[2j]}^{(p)}(w, u) \bar{P}_{[2j]}^\mu(w, u) \right), \\
\int \frac{d^4 l}{(2\pi)^4} l^\mu l^\nu \Delta\mathcal{G}(l - \tfrac{1}{2}w; w, u) &
\end{aligned}$$

$$= \sum_{i,j=3}^8 \Delta J_{[ij]}^{(p)}(w, u) P_{[ij]}^{\mu\nu}(w, u) + \sum_{i,j=1}^2 \Delta J_{[ij]}^{(q)}(w, u) Q_{[ij]}^{\mu\nu}(w, u) ,$$

$$\Delta \mathcal{G}(l - \frac{1}{2}w; w, u) = -i \left(\mathcal{S}_N(l, u) \mathcal{D}_K(w - l, u) - S_N(l) D_K(w - l) \right) , \quad (20)$$

which can be decomposed in terms of our projectors. The reduced loop functions $\Delta J_{[ij]}(w, u)$ acquire the generic form

$$\Delta J_{[ij]}(w, u) = \int \frac{d^4 l}{(2\pi)^4} g(l; w, u) \Delta J_{[ij]}(l; w, u) ,$$

$$g(l; w, u) = -2\pi \Theta(l \cdot u) \delta(l^2 - m_N^2) \frac{\Theta(k_F^2 + m_N^2 - (u \cdot l)^2)}{(w - l)^2 - m_K^2 - \Pi_{\bar{K}}(w - l, u)}$$

$$- \frac{i}{l^2 - m_N^2 + i\epsilon} \frac{1}{(w - l)^2 - m_K^2 - \Pi_{\bar{K}}(w - l, u)}$$

$$+ \frac{i}{l^2 - m_N^2 + i\epsilon} \frac{1}{(w - l)^2 - m_K^2 + i\epsilon} , \quad (21)$$

where the scalar polynomials $\Delta J_{[ij]}(l; w, u)$ involve typically some powers of $l^2, l \cdot w$ or $l \cdot u$. They are listed in Appendix B. We observe that the reduced loop functions $\Delta J_{[ij]}(w, u)$ are scalar and therefore depend only on w^2 and $w \cdot u$. Thus the loop functions can be evaluated in any convenient frame without loss of information. In practice we perform the loop integration in the rest frame of nuclear matter with $u_\mu = (1, \vec{0})$. It is straightforward to perform the energy and azimuthal angle integration in (21). The energy integration of the last two terms in (21) is performed by closing the complex contour in the upper complex half plane. One picks up two contributions: first the nucleon pole leading to $l_0 = \sqrt{m_N^2 + \vec{l}^2}$ in (21) and second the kaon pole at typically $w_0 - l_0 = -\sqrt{m_K^2 + (\vec{l} - \vec{w})^2}$. Here we neglect the kaon pole contribution since it is expected to be small due to the only moderate change of the kaon spectral function in nuclear matter: for the kaon pole contribution the second and third term in (21) cancel to good accuracy. All together one is left with a two-dimensional integral which must be evaluated numerically. In our numerical simulation we restrict $|\vec{l}| < 800$ MeV as to avoid poorly controlled contributions from the antikaon-nucleon scattering amplitudes for energies larger than about $\sqrt{s} \simeq 1700$ MeV.

Note that the center of mass frame of the antikaon-nucleon system with $w_\mu = (\sqrt{s}, \vec{0})$ does not necessarily lead to any further simplification as suggested in [15] since in this frame a nonzero bulk velocity $\vec{v} \neq 0$ is required. After performing the energy and azimuthal angle integration in (21) one is left with a two dimensional integration which must be evaluated numerically. The scalar self energy $\Pi_{\bar{K}}(l - w, u)$ depends on the two invariants $(l - w)^2$ and $(l - w) \cdot u$ only. Consequently in the nuclear matter rest frame the first entry involves

the angle $\vec{l} \cdot \vec{w}$ as compared to the frame with $w_\mu = (\sqrt{s}, \vec{0})$ which leads the angle $\vec{l} \cdot \vec{v}$ in second entry. This confirms the expected conservation of complexity. A strong 'vector' potential in the antikaon self energy, as suggested by phenomenology, actually means a strong dependence on $(l - w) \cdot u$ in (21). Therefore it appears unjustified to neglect that dependence as was done in [15,21,13].

Finally the in-medium Bethe-Salpeter equation reduces to a simple matrix equation

$$\begin{aligned}\mathcal{M}_{[ij]}^{(p)}(w, u) &= M_{[ij]}^{(p)}(\sqrt{s}) + \sum_{l,k=1}^8 M_{[ik]}^{(p)}(\sqrt{s}) \Delta J_{[kl]}^{(p)}(w, u) \mathcal{M}_{[lj]}^{(p)}(w, u) , \\ \mathcal{M}_{[ij]}^{(q)}(w, u) &= M_{[ij]}^{(q)}(\sqrt{s}) + \sum_{l,k=1}^2 M_{[ik]}^{(q)}(\sqrt{s}) \Delta J_{[kl]}^{(q)}(w, u) \mathcal{M}_{[lj]}^{(q)}(w, u) ,\end{aligned}\quad (22)$$

where $s = w_0^2 - \vec{w}^2$. We identify the nonzero vacuum matrix elements $M_{[ij]}^{(p,q)}(\sqrt{s})$ by

$$\begin{aligned}M_{[11]}^{(q)}(\sqrt{s}) &= -3 M^{(+)}(\sqrt{s}, 1) , \quad M_{[22]}^{(q)}(\sqrt{s}) = 3 M^{(-)}(\sqrt{s}, 1) , \\ M_{[11]}^{(p)}(\sqrt{s}) &= M^{(+)}(\sqrt{s}, 0) , \quad M_{[22]}^{(p)}(\sqrt{s}) = -M^{(-)}(\sqrt{s}, 0) , \\ M_{[77]}^{(p)}(\sqrt{s}) &= -3 M^{(+)}(\sqrt{s}, 1) , \quad M_{[88]}^{(p)}(\sqrt{s}) = 3 M^{(-)}(\sqrt{s}, 1) .\end{aligned}\quad (23)$$

Note that in (23) the isospin index I is suppressed. We observe that in fact all invariant amplitudes $\mathcal{M}_{[ij]}^{(p)}(w, u)$ with either $i \in \{3, 4, 5, 6\}$ or $j \in \{3, 4, 5, 6\}$ are zero. That is a direct consequence of the analogous property for the vacuum amplitudes $M_{[ij]}$. We emphasize that in principle the vacuum scattering amplitudes could have contributions proportional to $P_{[33]}, P_{[44]}, P_{[55]}$ or $P_{[66]}$ also. The reason why we do not consider such terms is that they are not determined by the on-shell antikaon-nucleon scattering process. The size of such terms merely reflects the choice of the interpolating fields [28,29] and therefore were not determined in [26]. They acquire physical significance only, if at the same time the free-space 3-body scattering processes are evaluated systematically. The antikaon self energy follows

$$\begin{aligned}\Pi_{\bar{K}}(q, u) &= - \sum_{i,j=1}^8 \int_0^{k_F} \frac{d^3 p}{(2\pi)^3} \frac{2}{E_p} c_{[ij]}^{(p)}(q; w, u) \bar{\mathcal{M}}_{[ij]}^{(p)}(w, u) \\ &\quad - \sum_{i,j=1}^2 \int_0^{k_F} \frac{d^3 p}{(2\pi)^3} \frac{2}{E_p} c_{[ij]}^{(q)}(q; w, u) \bar{\mathcal{M}}_{[ij]}^{(q)}(w, u) ,\end{aligned}\quad (24)$$

where we used $u_\mu = (1, \vec{0})$, $q_\mu = (\omega, \vec{q})$ and $w_\mu = (\omega + E_p, \vec{q} + \vec{p})$ with

$E_p = (m_N^2 + \vec{p}^2)^{1/2}$. The scalar coefficient functions $c_{[ij]}^{(p,q)}(q; w, u)$ are listed in Appendix C. In (24) we introduced the isospin averaged amplitudes:

$$\bar{\mathcal{M}}_{[ij]}(w, u) = \frac{1}{4} \mathcal{M}_{[ij]}^{(I=0)}(w, u) + \frac{3}{4} \mathcal{M}_{[ij]}^{(I=1)}(w, u). \quad (25)$$

With (24), (23) and (21) we arrive at our final self consistent set of equations which is solved numerically by iteration. First one determines the leading antikaon self energy $\Pi_{\bar{K}}(\omega, \vec{q})$ by (24) with $\mathcal{M}_{[ij]} = M_{[ij]}$. That leads via the loop functions (21) and the in-medium Bethe-Salpeter equation (23) to medium modified scattering amplitudes $\mathcal{M}_{[ij]}(w_0, \vec{w})$. The latter are used to determine the antikaon self energy of the next iteration. This procedure typically converges after 3 to 4 iterations. The manifest covariant form of the self energy and scattering amplitudes are recovered with $\Pi_{\bar{K}}(q^2, \omega) = \Pi_{\bar{K}}(q^2, q \cdot u)$ and $\mathcal{M}_{[ij]}(w^2, w_0) = \mathcal{M}_{[ij]}(w^2, w \cdot u)$ in a straightforward manner if considered as functions of q^2, ω and w^2, w_0 respectively.

4 Hyperon resonances in nuclear matter

We give a presentation and discussion of our results for the in-medium modification of the hyperon resonance properties. The resonance propagator may be identified with the appropriate antikaon-nucleon scattering amplitude of a given partial wave. In the self consistent scheme of section 3 the in-medium scattering process is intimately related to the antikaon spectral function, for which our results will be presented in the next section. According to (24) once the self consistent in-medium scattering process is established the antikaon self energy follows by averaging the in-medium scattering amplitudes over the Fermi distribution (24). Therefore the pertinent structures in the in-medium amplitudes already tell the characteristic features expected in the antikaon spectral function. The χ -BS(3) approach of [26], applied here to the nuclear antikaon dynamics, encounter many hyperon resonances. Firmly established are the s-wave $\Lambda(1405)$, the p-wave $\Sigma(1385)$ and the d-wave $\Lambda(1520)$ resonances, for all of which we derive their in-medium properties. Less reliably established are the s-wave $\Lambda(1800)$, the p-wave $\Lambda(1600)$ and the two d-wave $\Sigma(1690)$ and $\Lambda(1680)$ resonances because their properties were not directly constrained by the empirical data set. It was a highly nontrivial but expected result of [26], that all resonances but the p-wave baryon decuplet and the d-wave baryon nonet resonances were generated dynamically by the chiral coupled-channel dynamics once agreement with the low-energy data set with $p_{\text{lab.}} < 500$ MeV was achieved. A more accurate description of the latter resonances requires the extension of the χ -BS(3) approach by including more inelastic channels.

A particularly interesting and novel phenomenon observed in our approach is the in-medium induced mixing of resonances with different quantum numbers J^P . In a given isospin channel we find that at vanishing three momentum of the meson-baryon state $|\vec{w}| = 0$, the four partial wave amplitudes considered in this work, group into three different channels. Here it is convenient to associate with a given partial wave amplitude $L_{2I,2J}$ a sign ' \pm ' where the ' \pm ' refers to the way the angular momentum L determines the total angular momentum J with $J = L \pm \frac{1}{2}$. In this notation we find that at zero momentum the '+' amplitudes couple to the '-' amplitudes only. That implies for instance that either the S_{01} and D_{03} , the P_{01} and P_{03} or the P_{03} and D_{03} amplitudes influence each other pairwise. Once the meson-baryon pair is moving relative to the nuclear matter bulk with $|\vec{w}| \neq 0$ we find two separate channels only. Now all states with the same total angular momentum J start to interact also. Hence one finds that in the first channel, our P-space, all considered partial wave amplitudes S_{01} , P_{01} , P_{03} and D_{03} couple but in the second channel, our Q-space, only the partial wave amplitudes P_{03} and D_{03} do. The fact that the $J = \frac{3}{2}$ amplitudes P_{03} and D_{03} affect both the P- and Q-space is not surprising, because for those states one would expect an in-medium splitting of their four modes which are degenerate in free-space. This is somewhat analogous to the longitudinal and transverse modes of vector-mesons, which also split in nuclear matter.

We summarize that one expects sizeable effects from the in-medium mixing of the partial-wave amplitudes if both the S_{01} and D_{03} , or both the P_{01} and P_{03} or both the P_{03} and D_{03} amplitudes show significant strength at a given energy w_0 . To some extent this is the case for the amplitudes reflecting the $\Sigma(1195)$ ground state and the p-wave $\Sigma(1385)$ resonance for which we anticipate important mixing effects at large densities. At intermediate nuclear densities more pronounced phenomena may follow from the momentum induced mixing. Whereas at zero momentum of the hyperons with $|\vec{w}| = 0$ the mixing is suppressed by some powers of the Fermi momentum k_F^2 , at finite momentum it may grow with $|\vec{w}|^2$. To conclude we would predict that a $\Lambda(1115)$ moving relative to the nuclear matter bulk may show an interesting interplay with the s-wave $\Lambda(1405)$ resonance.

In Fig. 1 we present our results for the in-medium propagation of the $\Lambda(1115)$ ground state, the s-wave $\Lambda(1405)$ and the d-wave $\Lambda(1520)$ resonance all at nuclear saturation density $\rho_0 = 0.17 \text{ fm}^{-3}$. A $\Lambda(1115)$ with zero momentum $\vec{w} = 0$ is described well by a quasi-particle state with an attractive energy shift of about 30 MeV and a small decay width of about 5 MeV. However, as shown in Fig. 1 already at a three-momentum of $|\vec{w}| = 400 \text{ MeV}$ the $\Lambda(1115)$ receives a large width of about 20 MeV and is no longer accurately represented by a quasi-particle peak. That dramatic effect we interpret as an important mixing with the s-wave $\Lambda(1405)$ resonance. At $|\vec{w}| = 0$ the s-wave resonance by itself is considerably broadened and subject to an attractive energy shift of about 130 MeV. Here we find a considerably stronger in-medium effect for

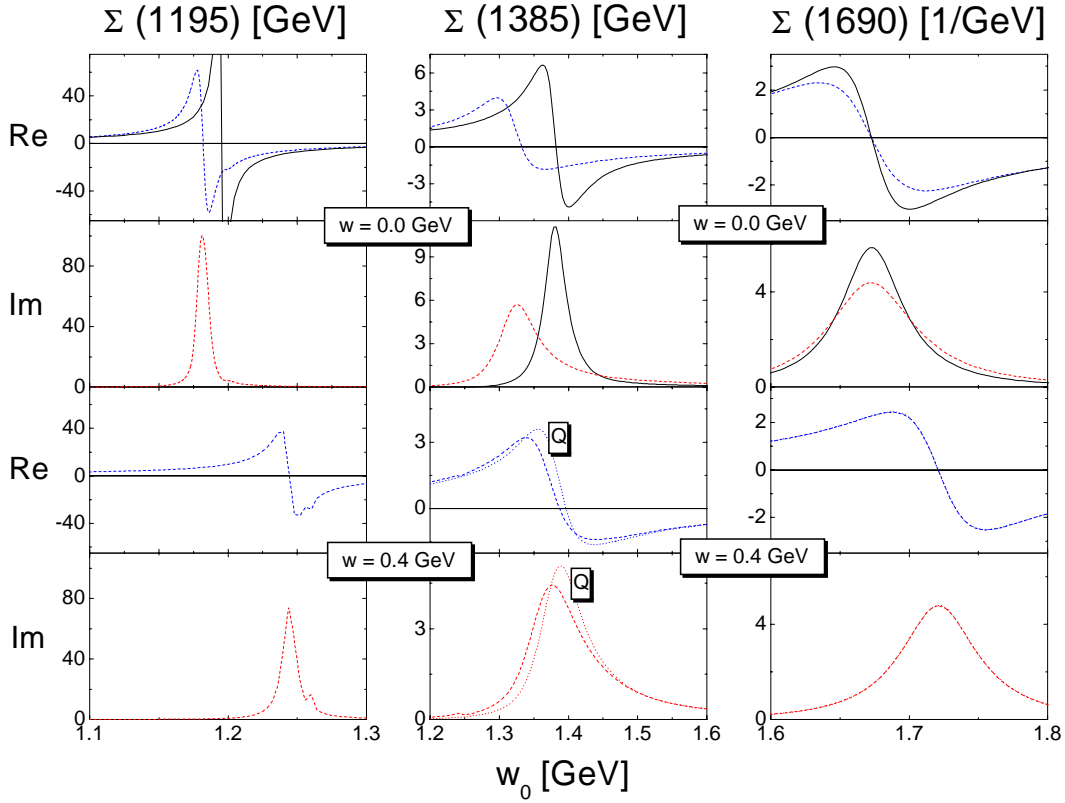


Fig. 2. In-medium properties of Σ hyperons with energy w_0 and momentum \vec{w} at nuclear saturation density $\rho = 0.17 \text{ fm}^{-3}$. The amplitudes plotted with dashed lines are $-f^2 M_{[22]}^{(p)}$ for the $\Sigma(1115)$, $-f^2 M_{[77]}^{(p)}/3$, $-f^2 M_{[11]}^{(q)}/3$ for the $\Sigma(1385)$ and $f^2 M_{[88]}^{(p)}/3$, $f^2 M_{[22]}^{(q)}/3$ for the $\Sigma(1690)$ where $f = 90 \text{ MeV}$ (see (23)). The solid lines give the corresponding amplitudes in the free-space limit with $\rho = 0$.

0 reflects simply the larger phase available for the in-medium decay $\Sigma \rightarrow \bar{K}N$. Interesting in-medium modifications are also observed for the $\Sigma(1385)$ resonance with an attractive mass shift of about 60 MeV and an decay width increased to about 70 MeV. Again a free space dispersion relation predicts well the quasi-particle energy of the $\Sigma(1385)$ resonance at $|\vec{w}| \neq 0$. Moreover the P- and Q-modes of that $J = \frac{3}{2}$ resonance are almost degenerate with about only 10 MeV less attraction for the Q-mode. The only hyperon resonance for which we do not predict sizeable in-medium modifications in our present scheme is the d-wave $\Sigma(1690)$ resonance. As seen in Fig. 2 that resonance is very little affected by its nuclear environment. That may simply reflect that the $\Sigma(1690)$ resonance has a rather small branching fraction of about 10 % into the $\bar{K}N$ channel only, even though the available phase space for that channel is large.

It may be worth giving a complementary interpretation for the strong in-medium effects established here and thereby anticipate part of our results for the antikaon spectral function. The medium modification of a given hyperon state is determined by its interaction with the nucleons. Our self consistent

approach includes in particular the t-channel antikaon exchange contribution for that interaction. A dramatic in-medium modification of the antikaon spectral function as will be established below, then necessarily leads to strong effects also for the hyperon states. For instance if the antikaon spectral function shows strength at energies $\omega < m_\Lambda - m_N$ the $\Lambda(1115) \rightarrow \bar{K}N$ may decay into a nucleon and an effective antikaon-like mode giving it a finite hadronic decay width.

5 Antikaon spectral function in nuclear matter

We carefully established a microscopic theory for the propagation of antikaons in cold nuclear matter. Of central importance for the derivation of a realistic antikaon spectral function are the subthreshold $\bar{K}N$ scattering amplitudes well established by the χ -BS(3) approach [26]. Since the antikaon spectral function tests the $\bar{K}N$ amplitudes at subthreshold energies where they are not directly determined by empirical data it is crucial to take $\bar{K}N$ amplitudes as input for the many-body calculation which are consistent with constraints set by causality, chiral symmetry and crossing symmetry. We therefore believe that the amplitudes of the χ -BS(3) approach are best suited for an application to the nuclear antikaon dynamics.

In Fig. 3 we present the antikaon spectral function evaluated at nuclear densities ρ_0 and $2\rho_0$ according to various approximation strategies. The spectral functions exhibit a rich structure with a pronounced dependence on the antikaon three-momentum. That reflects the presence of the various hyperon states in the $\bar{K}N$ amplitudes. Typically the peaks seen are quite broad and not always of quasi-particle type. In the first and third row the antikaon self energy is computed in terms of the free-space scattering amplitudes only. Here the first row gives the result with only s-wave contributions and the third row includes all s-, p- and d-wave contributions established in this work. The second and fourth row give results obtained in the self consistent approach developed in section 3 where the full result of the last row includes all partial waves and the results in the second row follow with s-wave contributions only. We observe that in all cases a self consistent evaluation of the spectral function leads to dramatic changes in the spectral function as compared to a calculation which is based on the free-space scattering amplitudes only. Furthermore we assure that the sum rule,

$$-\int \frac{d\omega}{\pi} \omega \Im S_{\bar{K}}(\omega, \vec{q}) = 1, \quad (26)$$

is satisfied to good accuracy. A violation of that sum rule (26) would indicate that the microscopic theory is in conflict with constraints set by causality.

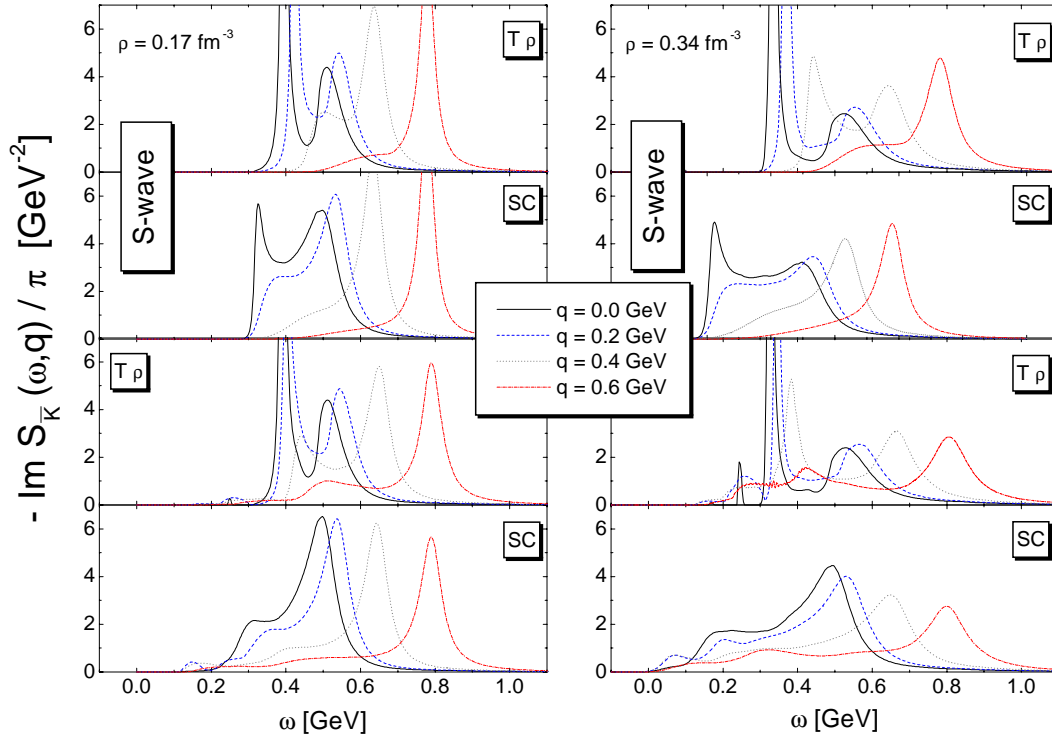


Fig. 3. Antikaon spectral function as a function of antikaon energy ω and momentum q . The labels 'T ρ ' and 'SC' refer to calculations obtained in terms of free-space and in-medium $\bar{K}N$ amplitudes respectively (see (9)). The first two rows give the results with only s-wave interactions and the last two rows with all s-, p- and d-wave contributions.

The fact that we find considerable more attraction with s-wave input only, as compared to previous works [14,15] reflects quantitative differences in the free-space $\bar{K}N$ dynamics. As is evident upon comparing the first and third rows of Fig. 3 the p- and d-wave contributions add quite significant attraction for small energies and large momenta. At about twice nuclear saturation density we find most striking the considerable support of the spectral function at zero energy. That reflects the coupling of the antikaon to a $\Lambda(1115)$ nucleon-hole state. Upon inspecting the Lindhard function of that contribution for a free-space $\Lambda(1115)$ [18] one finds that the antikaon spectral function should be non-zero for

$$\omega > \sqrt{m_\Lambda^2 + (|\vec{q}| - k_F)^2} - \sqrt{m_N^2 + k_F^2}. \quad (27)$$

The strength of the spectral function is largest at $|\vec{q}| \sim k_F$ because of the kinematical consideration (27). The non-zero support at $\omega = 0$ already present at $2\rho_0$ is a consequence of a substantial broadening and a large attractive energy shift for the $\Lambda(1115)$ state. That explains why the effects induced by the $\Lambda(1115)$ hyperon exchange established here are significantly larger than suggested in previous work [18]. Strength in the antikaon spectral function at

zero energy does not necessarily imply an antikaon condensation mechanism because the established zero modes have a finite hadronic decay width. Also the results at $2\rho_0$ should be considered cautiously because nuclear binding and correlation effects were not yet included in the present scheme.

6 Summary and outlook

The microscopic χ -BS(3) dynamics of [26] was applied to antikaon and hyperon resonance propagation in cold nuclear matter in a manifest covariant and self consistent manner. Of central importance for the microscopic evaluation of the antikaon spectral function in nuclear matter are the antikaon-nucleon scattering amplitudes, in particular at subthreshold energies. The required amplitudes were well established by the χ -BS(3) approach and show sizeable contributions from p-waves not considered systematically so far [18,14,15]. For the antikaon spectral function we predict a pronounced dependence on the three-momentum of the antikaon. The spectral function shows typically a rather wide structure invalidating a simple quasi-particle description. For instance at $\rho = 0.17 \text{ fm}^{-3}$ the strength starts at the quite small energy $\omega \simeq 100 \text{ MeV}$. At nuclear saturation density we predict attractive mass shifts for the $\Lambda(1405)$, $\Sigma(1385)$ and $\Lambda(1520)$ of about 130 MeV, 60 MeV and 100 MeV respectively. The hyperon states are found to show at the same time an increased decay width of about 150 MeV for the s-wave $\Lambda(1405)$, 70 MeV for the p-wave $\Sigma(1385)$ and 100 MeV for the d-wave $\Lambda(1520)$ resonances. Furthermore the attractive mass shifts for the Λ hyperon ground states of about 30 MeV is quite compatible with phenomenological constraints set by hyper-nuclei [30] considering the fact that we obtain a strong non-linear dependence of the hyperon masses on the nuclear density. It is less clear how to evaluate the consistency of the attractive mass shift of about 15 MeV found for the $\Sigma(1195)$ hyperon. Phenomenological constraints set by Σ^- -atom data are still inconclusive at present [31]. Most striking is the dramatic dependence of the in-medium decay width of the $\Lambda(1115)$ on its three-momentum derived in this work. At momentum 400 MeV the width of the $\Lambda(1115)$ is found to be about 20 MeV as compared to its width at zero momentum of about 5 MeV. We interpret this phenomenon as an in-medium mixing of the $\Lambda(1115)$ and $\Lambda(1405)$ states possible only at finite momentum. A complementary more microscopic explanation follows in terms of the t-channel antikaon exchange contribution dominating the hyperon-nucleon interaction. The strong density dependence of the antikaon spectral function, derived in this work, with sizeable support at small energies and a significant dependence on the antikaon momentum leads to the non-trivial propagation properties of the $\Lambda(1115)$ state. It is interesting to speculate to what extent this novel phenomenon has consequences for the formation of Λ -nuclei typically described so far in terms of effective mean field

models. That may deserve further detailed studies. Also it would be useful to explore whether it is feasible to confirm the strong in-medium modifications of hyperon states suggested in this work by suitable experiments at ELSA or MAMI.

We expect our analysis to pave the way for a microscopic description of kaonic atom data. The latter are known to be a rather sensitive test of the antikaon-nucleon dynamics [1,2]. An extended domain of applicability for the microscopic chiral theory is foreseen once additional inelastic channels are considered systematically. Also an application of the covariant and self consistent many-body dynamics, developed in this work, to the propagation properties of pions, vector-mesons and nucleon resonances is being pursued. Here we anticipate interesting in-medium effects due to the mixing of the s-, p- and d-wave nucleon resonances. Before an application of our results to heavy-ion reactions it is necessary to extend our many-body framework to finite temperature.

Acknowledgments

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7 Appendix A: Projector algebra

In order to construct the desired projector algebra $P_{[ij]}$ and $Q_{[ij]}$ of (17) it is useful to find appropriate building blocks which greatly facilitate the derivation. We introduce the objects P_{\pm} , U_{\pm} , V_{μ} and L_{μ} , R_{μ} :

$$\begin{aligned}
P_{\pm}(w) &= \frac{1}{2} \left(1 \pm \frac{\psi}{\sqrt{w^2}} \right), \quad U_{\pm}(w, u) = P_{\pm}(w) \frac{i \gamma \cdot u}{\sqrt{(w \cdot u)^2/w^2 - 1}} P_{\mp}(w), \\
V_{\mu}(w) &= \frac{1}{\sqrt{3}} \left(\gamma_{\mu} - \frac{\psi}{w^2} w_{\mu} \right), \quad X_{\mu}(w, u) = \frac{(w \cdot u) w_{\mu} - w^2 u_{\mu}}{w^2 \sqrt{(w \cdot u)^2/w^2 - 1}}, \\
R_{\mu}(w, u) &= -\frac{1}{\sqrt{2}} \left(U_{+}(w, u) + U_{-}(w, u) \right) V_{\mu}(w) - i \sqrt{\frac{3}{2}} X_{\mu}(w, u), \\
L_{\mu}(w, u) &= -\frac{1}{\sqrt{2}} V_{\mu}(w) \left(U_{+}(w, u) + U_{-}(w, u) \right) - i \sqrt{\frac{3}{2}} X_{\mu}(w, u), \quad (28)
\end{aligned}$$

which enjoy the following properties:

$$P_{\pm} P_{\pm} = P_{\pm} = U_{\pm} U_{\mp}, \quad P_{\pm} P_{\mp} = 0 = U_{\pm} U_{\pm},$$

$$\begin{aligned}
V \cdot L = 0 = R \cdot V, \quad L \cdot V = -\frac{\sqrt{8}}{3} (U_+ + U_-) = V \cdot R, \\
V \cdot V = L \cdot R = R \cdot L = 1, \quad R \cdot R = L \cdot L = \frac{1}{3}, \\
P_\pm V_\mu = V_\mu P_\mp, \quad P_\pm L_\mu = L_\mu P_\pm, \quad P_\pm R_\mu = R_\mu P_\pm, \\
U_\pm V_\mu = -\frac{1}{3} V_\mu U_\mp - \frac{\sqrt{8}}{3} L_\mu P_\mp, \quad U_\pm L_\mu = R_\mu U_\pm, \\
V_\mu U_\pm = -\frac{1}{3} U_\mp V_\mu - \frac{\sqrt{8}}{3} R_\mu P_\mp, \quad U_\pm R_\mu = L_\mu U_\pm.
\end{aligned} \tag{29}$$

The projectors are:

$$\begin{aligned}
P_{[11]} &= P_+, \quad P_{[12]} = U_+, \quad P_{[21]} = U_-, \quad P_{[22]} = P_-, \\
P_{[31]}^\mu &= V_\mu P_+, \quad P_{[32]}^\mu = V_\mu U_+, \quad \bar{P}_{[13]}^\mu = P_+ V^\mu, \quad \bar{P}_{[23]}^\mu = U_- V^\mu, \\
P_{[41]}^\mu &= V_\mu U_-, \quad P_{[42]}^\mu = V_\mu P_-, \quad \bar{P}_{[14]}^\mu = U_+ V^\mu, \quad \bar{P}_{[24]}^\mu = P_- V^\mu, \\
P_{[51]}^\mu &= \hat{w}_\mu P_+, \quad P_{[52]}^\mu = \hat{w}_\mu U_+, \quad \bar{P}_{[15]}^\mu = P_+ \hat{w}^\mu, \quad \bar{P}_{[25]}^\mu = U_- \hat{w}^\mu, \\
P_{[61]}^\mu &= \hat{w}_\mu U_-, \quad P_{[62]}^\mu = \hat{w}_\mu P_-, \quad \bar{P}_{[16]}^\mu = U_+ \hat{w}^\mu, \quad \bar{P}_{[26]}^\mu = P_- \hat{w}^\mu, \\
P_{[71]}^\mu &= L_\mu P_+, \quad P_{[72]}^\mu = L_\mu U_+, \quad \bar{P}_{[17]}^\mu = P_+ R^\mu, \quad \bar{P}_{[27]}^\mu = U_- R^\mu, \\
P_{[81]}^\mu &= L_\mu U_-, \quad P_{[82]}^\mu = L_\mu P_-, \quad \bar{P}_{[18]}^\mu = U_+ R^\mu, \quad \bar{P}_{[28]}^\mu = P_- R^\mu, \\
P_{[ij]}^{\mu\nu} &= P_{[i1]}^\mu \bar{P}_{[1j]}^\nu = P_{[i2]}^\mu \bar{P}_{[2j]}^\nu, \\
Q_{[11]}^{\mu\nu} &= (g^{\mu\nu} - \hat{w}^\mu \hat{w}^\nu) P_+ - V^\mu P_- V^\nu - L^\mu P_+ R^\nu, \\
Q_{[22]}^{\mu\nu} &= (g^{\mu\nu} - \hat{w}^\mu \hat{w}^\nu) P_- - V^\mu P_+ V^\nu - L^\mu P_- R^\nu, \\
Q_{[12]}^{\mu\nu} &= (g^{\mu\nu} - \hat{w}^\mu \hat{w}^\nu) U_+ + \frac{1}{3} V^\mu U_- V^\nu \\
&\quad + \frac{\sqrt{8}}{3} (L^\mu P_+ V^\nu + V^\mu P_- R^\nu) - \frac{1}{3} L^\mu U_+ R^\nu, \\
Q_{[21]}^{\mu\nu} &= (g^{\mu\nu} - \hat{w}^\mu \hat{w}^\nu) U_- + \frac{1}{3} V^\mu U_+ V^\nu \\
&\quad + \frac{\sqrt{8}}{3} (L^\mu P_- V^\nu + V^\mu P_+ R^\nu) - \frac{1}{3} L^\mu U_- R^\nu,
\end{aligned} \tag{30}$$

where $\hat{w}_\mu = w_\mu / \sqrt{w^2}$. By means of (29) it is straightforward to verify (17).

8 Appendix B: Evaluation of loop functions

The loop matrix $\Delta J_{[ij]}^{(p)}(w, u)$ and $\Delta J_{[ij]}^{(q)}(w, u)$ can be expressed in terms of 13 independent loop functions $\Delta J_{i=0,\dots,12}(w, u)$

$$\Delta J_i(w, u) = \int \frac{d^4 l}{(2\pi)^4} g(l; w, u) \Delta J_i(l; w, u), \tag{31}$$

and

$$\begin{aligned}
\Delta J_0(l; w, u) &= 1, \quad \Delta J_1(l; w, u) = (l \cdot \hat{w}), \quad \Delta J_2(l; w, u) = -(l \cdot X(w, u)), \\
\Delta J_3(l; w, u) &= \frac{1}{2} \left(l^2 - (l \cdot \hat{w})^2 + (l \cdot X(w, u))^2 \right), \\
\Delta J_4(l; w, u) &= (l \cdot \hat{w})^2, \quad \Delta J_5(l; w, u) = (l \cdot X(w, u))^2, \\
\Delta J_6(l; w, u) &= -(l \cdot X(w, u)) (l \cdot \hat{w}), \\
\Delta J_7(l; w, u) &= \frac{1}{2} \left(l^2 - (l \cdot \hat{w})^2 + (l \cdot X(w, u))^2 \right) (l \cdot \hat{w}), \\
\Delta J_8(l; w, u) &= -\frac{1}{2} \left(l^2 - (l \cdot \hat{w})^2 + (l \cdot X(w, u))^2 \right) (l \cdot X(w, u)), \\
\Delta J_9(l; w, u) &= (l \cdot \hat{w})^3, \quad \Delta J_{10}(l; w, u) = -(l \cdot \hat{w})^2 (l \cdot X(w, u)), \\
\Delta J_{11}(l; w, u) &= -(l \cdot X(w, u))^3, \quad \Delta J_{12}(l; w, u) = (l \cdot X(w, u))^2 (l \cdot \hat{w}), \\
g(l; w, u) &= -2\pi \Theta(l \cdot u) \delta(l^2 - m_N^2) \frac{\Theta(k_F^2 + m_N^2 - (u \cdot l)^2)}{(l - w)^2 - m_K^2 - \Pi_K(w - l, u)} \\
&\quad - \frac{i}{l^2 - m_N^2 + i\epsilon} \frac{1}{(w - l)^2 - m_K^2 - \Pi_K(w - l, u)} \\
&\quad + \frac{i}{l^2 - m_N^2 + i\epsilon} \frac{1}{(w - l)^2 - m_K^2 + i\epsilon}, \tag{32}
\end{aligned}$$

where the 4-vector $X_\mu(w, u)$ was introduced in (28). The matrix elements are:

$$\begin{aligned}
\Delta J_{[11]}^{(q)} &= m_N \Delta J_3 + \Delta J_7, \quad \Delta J_{[22]}^{(q)} = m_N \Delta J_3 - \Delta J_7, \quad \Delta J_{[12]}^{(q)} = -i \Delta J_8, \\
\Delta J_{[11]}^{(p)} &= m_N \Delta J_0 + \Delta J_1, \quad \Delta J_{[22]}^{(p)} = m_N \Delta J_0 - \Delta J_1, \quad \Delta J_{[12]}^{(p)} = -i \Delta J_2, \\
\Delta J_{[13]}^{(p)} &= \Delta J_{[24]}^{(p)} = \frac{1}{\sqrt{3}} (2 \Delta J_3 - \Delta J_5), \quad \Delta J_{[16]}^{(p)} = \Delta J_{[25]}^{(p)} = -i \Delta J_6, \\
\Delta J_{[15]}^{(p)} &= m_N \Delta J_1 + \Delta J_4, \quad \Delta J_{[26]}^{(p)} = m_N \Delta J_1 - \Delta J_4, \\
\Delta J_{[17]}^{(p)} &= i \sqrt{\frac{2}{3}} (m_N \Delta J_2 + \Delta J_6), \quad \Delta J_{[28]}^{(p)} = i \sqrt{\frac{2}{3}} (m_N \Delta J_2 - \Delta J_6), \\
\Delta J_{[14]}^{(p)} &= -\frac{i}{\sqrt{3}} (m_N \Delta J_2 + \Delta J_6), \quad \Delta J_{[23]}^{(p)} = -\frac{i}{\sqrt{3}} (m_N \Delta J_2 - \Delta J_6), \\
\Delta J_{[18]}^{(p)} &= \Delta J_{[27]}^{(p)} = \sqrt{\frac{2}{3}} (\Delta J_3 + \Delta J_5), \tag{33}
\end{aligned}$$

and

$$\Delta J_{[33]}^{(p)} = \frac{1}{3} (m_N (2 \Delta J_3 - \Delta J_5) + \Delta J_{12} - 2 \Delta J_7),$$

$$\begin{aligned}
\Delta J_{[44]}^{(p)} &= \frac{1}{3} \left(m_N (2 \Delta J_3 - \Delta J_5) - \Delta J_{12} + 2 \Delta J_7 \right), \\
\Delta J_{[55]}^{(p)} &= m_N \Delta J_4 + \Delta J_9, \quad \Delta J_{[66]}^{(p)} = m_N \Delta J_4 - \Delta J_9, \\
\Delta J_{[77]}^{(p)} &= \frac{1}{3} \left(m_N (\Delta J_3 - 2 \Delta J_5) + \Delta J_7 - 2 \Delta J_{12} \right), \\
\Delta J_{[88]}^{(p)} &= \frac{1}{3} \left(m_N (\Delta J_3 - 2 \Delta J_5) - \Delta J_7 + 2 \Delta J_{12} \right), \\
\Delta J_{[35]}^{(p)} &= \Delta J_{[46]}^{(p)} = \frac{1}{\sqrt{3}} \left(2 \Delta J_7 - \Delta J_{12} \right), \\
\Delta J_{[37]}^{(p)} &= \Delta J_{[48]}^{(p)} = i \frac{\sqrt{2}}{3} \left(2 \Delta J_8 - \Delta J_{11} \right), \\
\Delta J_{[57]}^{(p)} &= i \sqrt{\frac{2}{3}} \left(m_N \Delta J_6 + \Delta J_{10} \right), \quad \Delta J_{[68]}^{(p)} = i \sqrt{\frac{2}{3}} \left(m_N \Delta J_6 - \Delta J_{10} \right), \\
\Delta J_{[34]}^{(p)} &= -\frac{i}{3} \left(2 \Delta J_8 - \Delta J_{11} \right), \quad \Delta J_{[78]}^{(p)} = \frac{i}{3} \left(5 \Delta J_8 + 2 \Delta J_{11} \right), \\
\Delta J_{[36]}^{(p)} &= -\frac{i}{\sqrt{3}} \left(m_N \Delta J_6 - \Delta J_{10} \right), \quad \Delta J_{[45]}^{(p)} = -\frac{i}{\sqrt{3}} \left(m_N \Delta J_6 + \Delta J_{10} \right), \\
\Delta J_{[38]}^{(p)} &= \frac{\sqrt{2}}{3} \left(m_N (\Delta J_3 + \Delta J_5) - \Delta J_7 - \Delta J_{12} \right), \\
\Delta J_{[47]}^{(p)} &= \frac{\sqrt{2}}{3} \left(m_N (\Delta J_3 + \Delta J_5) + \Delta J_7 + \Delta J_{12} \right), \\
\Delta J_{[58]}^{(p)} &= \Delta J_{[67]}^{(p)} = \sqrt{\frac{2}{3}} \left(\Delta J_7 + \Delta J_{12} \right), \quad \Delta J_{[56]}^{(p)} = -i \Delta J_{10}, \tag{34}
\end{aligned}$$

where the remaining elements follow from the symmetry property $\Delta J_{[ij]}^{(p,q)} = \Delta J_{[ji]}^{(p,q)}$.

9 Appendix C: Evaluation of kaon self energy

In the expression for antikaon self energy (9) the term

$$\begin{aligned}
&\frac{1}{2} \text{tr} \left[(\not{p} + m_N) \mathcal{T}_{\bar{K}N} \left(\frac{1}{2}(p - q), \frac{1}{2}(p - q); p + q, u \right) \right] \\
&= \sum_{i,j=1}^8 c_{[ij]}^{(p)}(q; w, u) \mathcal{M}_{[ij]}^{(p)}(w, u) + \sum_{i,j=1}^2 c_{[ij]}^{(q)}(q; w, u) \mathcal{M}_{[ij]}^{(q)}(w, u), \tag{35}
\end{aligned}$$

appears with $w = p + q$. The coefficient functions $c_{[ij]}^{(q)}(q; w, u)$ and $c_{[ij]}^{(p)}(q; w, u)$ are:

$$\begin{aligned}
c_{[11]}^{(q)} &= \frac{1}{2} E_+ (E_+ E_- + (X \cdot q)^2), \quad c_{[11]}^{(p)} = E_+, \\
c_{[12]}^{(q)} &= -\frac{i}{2} (X \cdot q) (E_+ E_- + (X \cdot q)^2), \quad c_{[12]}^{(p)} = -i (X \cdot q), \\
c_{[22]}^{(q)} &= \frac{1}{2} E_- (E_+ E_- + (X \cdot q)^2), \quad c_{[22]}^{(p)} = E_-, \\
c_{[13]}^{(p)} &= c_{[24]}^{(p)} = -\frac{1}{\sqrt{3}} E_+ E_-, \quad c_{[25]}^{(p)} = c_{[16]}^{(p)} = -i (\hat{w} \cdot q) (X \cdot q), \\
c_{[17]}^{(p)} &= -i \sqrt{\frac{2}{3}} E_+ (X \cdot q), \quad c_{[15]}^{(p)} = (\hat{w} \cdot q) E_+, \quad c_{[14]}^{(p)} = \frac{i}{\sqrt{3}} E_+ (X \cdot q), \\
c_{[28]}^{(p)} &= -i \sqrt{\frac{2}{3}} E_- (X \cdot q), \quad c_{[26]}^{(p)} = (\hat{w} \cdot q) E_-, \quad c_{[23]}^{(p)} = \frac{i}{\sqrt{3}} E_- (X \cdot q), \\
c_{[27]}^{(p)} &= c_{[18]}^{(p)} = -\sqrt{\frac{3}{2}} \left(\frac{1}{3} E_+ E_- + (X \cdot q)^2 \right), \tag{36}
\end{aligned}$$

and

$$\begin{aligned}
c_{[33]}^{(p)} &= \frac{1}{3} M_-^2 E_+, \quad c_{[44]}^{(p)} = \frac{1}{3} E_+^2 E_-, \\
c_{[55]}^{(p)} &= E_+ (\hat{w} \cdot q)^2, \quad c_{[77]}^{(p)} = \frac{1}{2} E_+ \left(\frac{1}{3} E_+ E_- - (X \cdot q)^2 \right), \\
c_{[66]}^{(p)} &= E_- (\hat{w} \cdot q)^2, \quad c_{[88]}^{(p)} = \frac{1}{2} E_- \left(\frac{1}{3} E_+ E_- - (X \cdot q)^2 \right), \\
c_{[35]}^{(p)} &= c_{[46]}^{(p)} = -\frac{1}{\sqrt{3}} (\hat{w} \cdot q) E_+ E_-, \quad c_{[57]}^{(p)} = -i \sqrt{\frac{2}{3}} (X \cdot q) (\hat{w} \cdot q) E_+, \\
c_{[37]}^{(p)} &= c_{[48]}^{(p)} = i \frac{\sqrt{2}}{3} (X \cdot q) E_+ E_-, \quad c_{[68]}^{(p)} = -i \sqrt{\frac{2}{3}} (X \cdot q) (\hat{w} \cdot q) E_-, \\
c_{[34]}^{(p)} &= -\frac{i}{3} (X \cdot q) E_+ E_-, \quad c_{[56]}^{(p)} = -i (\hat{w} \cdot q)^2 (X \cdot q), \\
c_{[78]}^{(p)} &= i (X \cdot q) \left(\frac{3}{2} (X \cdot q)^2 + \frac{5}{6} E_+ E_- \right), \\
c_{[36]}^{(p)} &= \frac{i}{\sqrt{3}} (\hat{w} \cdot q) E_- (X \cdot q), \quad c_{[38]}^{(p)} = \frac{1}{\sqrt{2}} E_- \left(\frac{1}{3} E_+ E_- + (X \cdot q)^2 \right), \\
c_{[45]}^{(p)} &= \frac{i}{\sqrt{3}} (\hat{w} \cdot q) E_+ (X \cdot q), \quad c_{[47]}^{(p)} = \frac{1}{\sqrt{2}} E_+ \left(\frac{1}{3} E_+ E_- + (X \cdot q)^2 \right), \\
c_{[58]}^{(p)} &= c_{[67]}^{(p)} = -\sqrt{\frac{3}{2}} (\hat{w} \cdot q) \left(\frac{1}{3} E_+ E_- + (X \cdot q)^2 \right), \tag{37}
\end{aligned}$$

where

$$E_{\pm} \equiv m_N \pm (\sqrt{w_0^2 - \vec{w}^2} - q \cdot \hat{w}), \quad E_+ E_- = q^2 - (q \cdot \hat{w})^2. \tag{38}$$

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